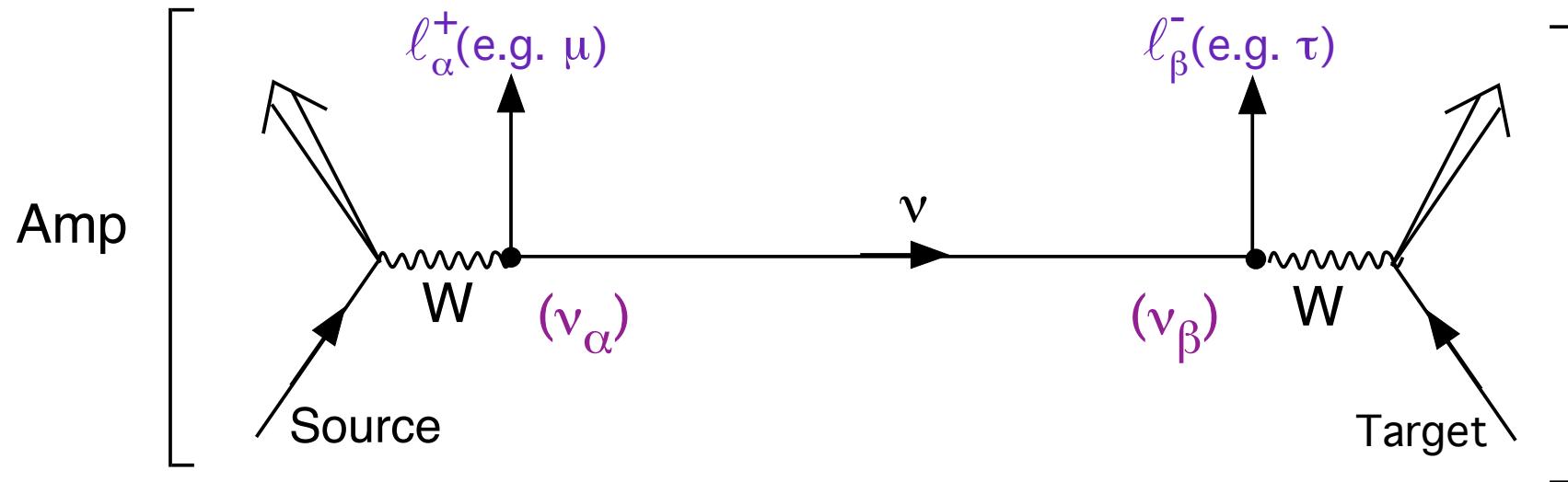


Neutrino Flavor Change (Oscillation)

in Vacuum

(Approach of B.K. & Stodolsky)



The diagram illustrates a neural network layer. On the left, a vertical bracket labeled "Source" contains multiple arrows pointing towards a central node. This node is labeled ℓ_{α}^+ above and $W U_{\alpha i}^*$ below. A horizontal arrow labeled v_i points to the right, representing the propagation of the signal. To the right of this arrow is the label "Prop(v_i)". Further to the right, another vertical bracket labeled "Target" contains multiple arrows pointing away from a central node. This node is labeled ℓ_{β}^- above and $U_{\beta i} W$ below. The entire expression is preceded by the equation $= \sum_i \text{Amp}$.

$$\text{Amp } [v_\alpha \rightarrow v_\beta] = \sum U_{\alpha i}^* \text{Prop}(v_i) U_{\beta i}$$

What is Propagator $(v_i) \equiv \text{Prop}(v_i)$?

In the v_i rest frame, where the proper time is τ_i ,

$$i \frac{\partial}{\partial \tau_i} |\nu_i(\tau_i)\rangle = m_i |\nu_i(\tau_i)\rangle \quad .$$

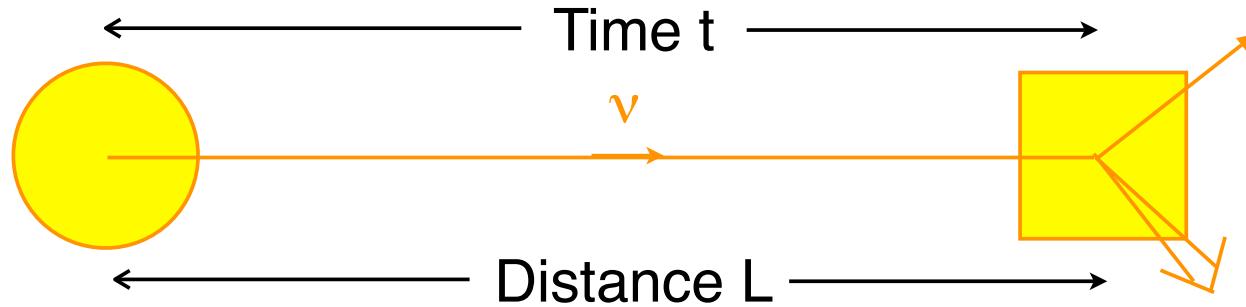
Thus,

$$|\nu_i(\tau_i)\rangle = e^{-im_i\tau_i} |\nu_i(0)\rangle \quad .$$

Then, the amplitude for propagation for time τ_i is —

$$\text{Prop}(\nu_i) \equiv \langle \nu_i(0) | \nu_i(\tau_i) \rangle = e^{-im_i\tau_i} \quad .$$

In the laboratory frame —



The experimenter chooses L and t .

They are common to all components of the beam.

For each v_i , by Lorentz invariance,

$$m_i \tau_i = E_i t - p_i L .$$

Neutrino sources are \sim constant in time.

Averaged over time, the

$$e^{-iE_1 t} - e^{-iE_2 t} \quad \text{interference}$$

is —

$$\langle e^{-i(E_1-E_2)t} \rangle_t = 0$$

unless $E_2 = E_1$.

Only neutrino mass eigenstates with a common energy E are coherent.

(Stodolsky)

For each mass eigenstate ,

$$p_i = \sqrt{E^2 - m_i^2} \cong E - \frac{m_i^2}{2E} .$$

Then the phase in the v_i propagator $\exp[-im_i\tau_i]$ is —

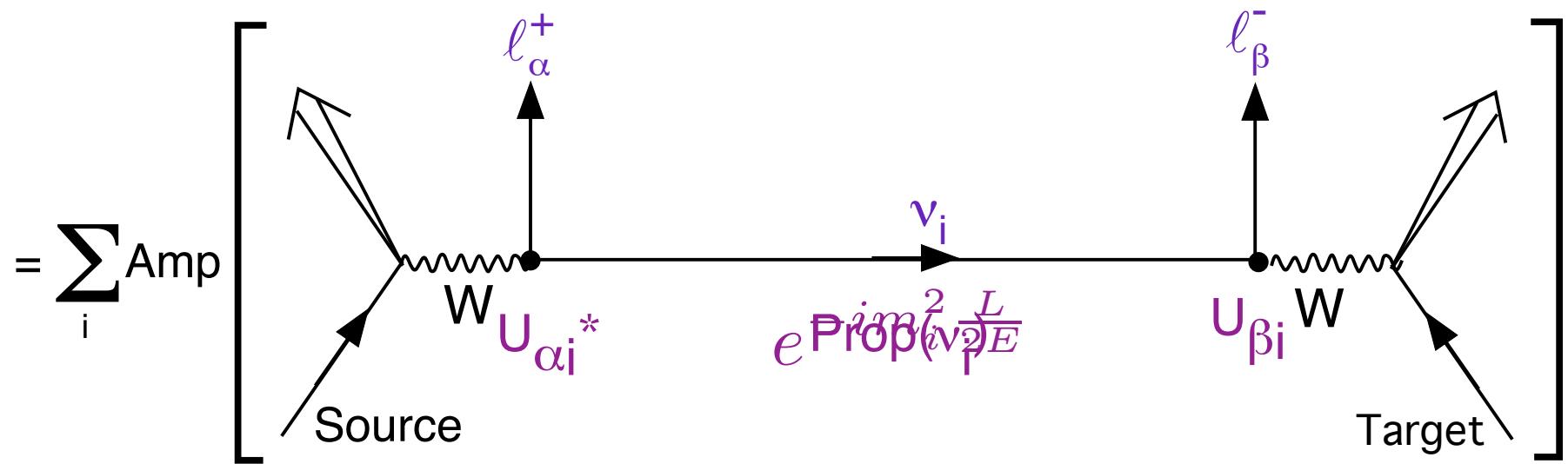
$$m_i\tau_i = E_i t - p_i L \cong Et - (E - m_i^2/2E)L$$

$$= E(t - L) + m_i^2 L / 2E .$$



Irrelevant overall phase \uparrow

Amp $[\nu_\alpha \rightarrow \nu_\beta]$



$$= \sum_i U_{\alpha i}^* e^{-im_i^2 \frac{L}{2E}} U_{\beta i}$$

Probability for Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\text{Amp}(\nu_\alpha \rightarrow \nu_\beta)|^2 =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2(\Delta m_{ij}^2 \frac{L}{4E})$$

$$+ 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin(\Delta m_{ij}^2 \frac{L}{2E})$$

where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$

For Antineutrinos –

We assume the world is CPT invariant.

Our formalism assumes this.

$$P(\overline{\nu_\alpha} \rightarrow \overline{\nu_\beta}) \stackrel{CPT}{=} P(\nu_\beta \rightarrow \nu_\alpha) = P(\nu_\alpha \rightarrow \nu_\beta; U \rightarrow U^*)$$

Thus,

$$P(\overset{\leftarrow}{\nu_\alpha} \rightarrow \overset{\leftarrow}{\nu_\beta}) =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2(\Delta m_{ij}^2 \frac{L}{4E})$$

$$\overset{+}{\leftarrow} 2 \sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin(\Delta m_{ij}^2 \frac{L}{2E})$$

A complex U would lead to the CP violation

$$P(\overline{\nu_\alpha} \rightarrow \overline{\nu_\beta}) \neq P(\nu_\alpha \rightarrow \nu_\beta) .$$